## MIT FIRST GRADE COLLEGE, Mysuru

## First Internals – IV B.COM/BBA ---July, 2021-2022

## Sub: Quantitative Techniques

Time: 90 mins Max Marks: 40

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### PART-A

**Answer any Three questions of the following** **(10X3 =30)**

**1**.Solve by using Cramer's rule

3x+2y+5z=32

2x+5y+3z=31

5x+3y+2z=27

**2.a**) A man occupies a post with the starting salary of RS 3,60,000 per annum, he get 10% increase in his salary every year for 5 years. What is his salary in the $5^{th}$ year and what is his total salary in the $1^{st}$ 5 years?

**b)** Find the sum of all natural numbers between 500 and 1000 which are divisible by 10?

**3.** If $A = \begin{pmatrix} 2 & 1 & 2 \\ 3 & 1 & 1 \\ 1 & 3 & 2 \end{pmatrix}$ and $B = \begin{pmatrix} -1 & 1 & 2 \\ 2 & -1 & 1 \\ 1 & 2 & 1 \end{pmatrix}$

Prove that (AB)' = B'A'

**4.** Find the 3 numbers in GP, such that their sum is 65 and their product is 3375

### PART-B

**Answer any Two of the following questions** **(5X2 =10)**

**5.** Find the $25^{th}$ term of the series 8+12+16+20

**6.** Which term of the sequence 3,6,12…….is 1536?

***PTO***

**7.** If $A = \begin{pmatrix} 8 & -7 & -3 \\ 4 & 17 & -11 \\ 13 & -9 & 2 \end{pmatrix}$ Find a) $A + A'$

b) $A' - A$

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********All THE BEST********

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Subject: QUANTITATIVE TECHNIQUES.

Course : B.B.M., B.Com., B.C.A., & M.Com.,

Marks Obtained 40/40

Sem : IV Sem | Sec : 'B'.

28/7/22.

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Part - B :

5. $8, 12, 16, 20 \ldots$

$a = 8$, $d = T_2 - T_1$, $T_{25} = ?$, $n = 25$.

$= 12 - 8$

$= 4.$

$T_n = a + (n-1)d.$

$T_{25} = 8 + (25-1)4.$

$T_{25} = 8 + 24 \times 4.$

$T_{25} = 8 + 96.$

5

$\therefore T_{25} = 104.$

The 25th term of the series $8+12+16+20 \ldots$ is 104.

6. $3, 6, 12 \ldots 1536.$

$a = 3$, $r = \frac{T_2}{T_1} = \frac{6}{3} = 2$, $T_n = 1536$, $n = ?$

$T_n = a \cdot r^{n-1}.$

$1536 = 3 \times (2)^{n-1}.$

$\frac{1536}{3} = (2)^{n-1}.$

$512 = (2)^{n-1}$

$(2)^9 = (2)^{n-1}$. [Since $(2)^9 = 512$].

5

$9 = n - 1$.

$\therefore n = 9 + 1 = \boxed{10}$.

$\therefore$ The 10th term of the series $3, 6, 12 \ldots$ is 1536.

Total

## Part - 'A'.

1. $3x+2y+5z=32$

$2x+5y+3z=31$

$5x+3y+2z=27.$

$$\begin{bmatrix} 3 & 2 & 5 \\ 2 & 5 & 3 \\ 5 & 3 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 32 \\ 31 \\ 27 \end{bmatrix}$$

$$\begin{array}{ccc} + & - & + \end{array}$$
$$\begin{bmatrix} 3 & 2 & 5 \\ 2 & 5 & 3 \\ 5 & 3 & 2 \end{bmatrix}$$

$$\Delta = 3\begin{vmatrix} 5 & 3 \\ 3 & 2 \end{vmatrix} - 2\begin{vmatrix} 2 & 3 \\ 5 & 2 \end{vmatrix} + 5\begin{vmatrix} 2 & 5 \\ 5 & 3 \end{vmatrix}$$

$= 3[(5\times2)-(3\times3)] - 2[(2\times2)-(3\times5)] + 5[(2\times3)-(5\times5)$

$= 3[10-9] - 2[4-15] + 5[6-25].$

$= 3(1) - 2(-11) + 5(-19).$

$= 3+22-95.$

$= 25-95$

$\boxed{\Delta = -70}$

$$\begin{array}{ccc} + & - & + \end{array}$$
$$\Delta x = \begin{bmatrix} 32 & 2 & 5 \\ 31 & 5 & 3 \\ 27 & 3 & 2 \end{bmatrix}$$

$$\Delta x = 32\begin{vmatrix} 5 & 3 \\ 3 & 2 \end{vmatrix} - 2\begin{vmatrix} 31 & 3 \\ 27 & 2 \end{vmatrix} + 5\begin{vmatrix} 31 & 5 \\ 27 & 3 \end{vmatrix}$$

$= 32[(5\times2)-(3\times3)] - 2[(31\times2)-(3\times27)] + 5[(31\times3)-(5$

$= 32[10+9] - 2[62-81] + 5[93-135].$

$= 32(1) - 2(-19) + 5(-42).$

$= 32 + 38 - 210.$

$= 70 - 210.$

$\boxed{\Delta x = -140}$

↳ $\Delta y = \begin{array}{ccc} + & - & + \end{array}$
$\begin{vmatrix} 3 & 32 & 5 \\ 2 & 31 & 3 \\ 5 & 27 & 2 \end{vmatrix}$

$\Delta y = 3\begin{vmatrix} 31 & 3 \\ 27 & 2 \end{vmatrix} - 32\begin{vmatrix} 2 & 3 \\ 5 & 2 \end{vmatrix} + 5\begin{vmatrix} 2 & 31 \\ 5 & 27 \end{vmatrix}.$

$= 3[(31\times2)-(3\times27)] - 32[(2\times2)-(3\times5)] + 5[(2\times27)-(31\times5)]$

$= 3[62-81] - 32[4-15] + 5[54-155].$

$= 3(-19) - 32(-11) + 5(-101).$

$= -57 + 352 - 505.$

$= 352 \cancel{325} - 562$

$\boxed{\Delta y = -210.}$

↳ $\Delta z = \begin{array}{ccc} + & - & + \end{array}$
$\begin{vmatrix} 3 & 2 & 32 \\ 2 & 5 & 31 \\ 5 & 3 & 27 \end{vmatrix}$

$\Delta z = 3\begin{vmatrix} 5 & 31 \\ 3 & 27 \end{vmatrix} - 2\begin{vmatrix} 2 & 31 \\ 5 & 27 \end{vmatrix} + 32\begin{vmatrix} 2 & 5 \\ 5 & 3 \end{vmatrix}.$

$= 3[(5\times27)-(31\times3)] - 2[(2\times27)-(31\times5)] + 32[(2\times3)-(5\times5)].$

$= 3[135-93] - 2[54-155] + 32[6-25].$

$= 3(42) - 2(-101) + 32(-19).$

$= 126 + 202 - 608.$

$= 328 - 608$

$\boxed{\Delta z = -280.}$

$$\therefore \quad x = \frac{\Delta x}{\Delta} = \frac{+140^{2}}{+70_{1}} = \boxed{2}.$$

$$y = \frac{\Delta y}{\Delta} = \frac{+210^{3}}{+70_{1}} = \boxed{3}.$$

$$z = \frac{\Delta z}{\Delta} = \frac{+280^{4}}{+70_{1}} = \boxed{4}.$$

$\therefore$ The 3 numbers $x, y, z$ are 2, 3 & 4 respectively

⊛ Verification:

Substituting $x, y$ & $z$ values in EQN ①.

$$3x + 2y + 5z = 32$$

$$3(2) + 2(3) + 5(4) = 32$$

$$6 + 6 + 20 = 32$$

$$32 = 32$$

LHS = RHS

3. $A = \begin{bmatrix} 2 & 1 & 2 \\ 3 & 1 & 1 \\ 1 & 3 & 2 \end{bmatrix}$ $\quad B = \begin{bmatrix} -1 & 1 & 2 \\ 2 & -1 & 1 \\ 1 & 2 & 1 \end{bmatrix}$

To prove: $(AB)' = B'A'$.

$\Rightarrow$ LHS: $(AB)'$.

$\hookrightarrow (A \times B) = \begin{bmatrix} 2 & 1 & 2 \\ 3 & 1 & 1 \\ 1 & 3 & 2 \end{bmatrix} \times \begin{bmatrix} -1 & 1 & 2 \\ 2 & -1 & 1 \\ 1 & 2 & 1 \end{bmatrix}$.

$$= \begin{bmatrix} (2\times-1)+(1\times2)+(2\times1) & (2\times1)+(1\times-1)+(2\times2) & (2\times2)+(1\times1)+(2\times1) \\ (3\times-1)+(1\times2)+(1\times1) & (3\times1)+(1\times-1)+(1\times2) & (3\times2)+(1\times1)+(1\times1) \\ (1\times-1)+(3\times2)+(2\times1) & (1\times1)+(3\times-1)+(2\times2) & (1\times2)+(3\times1)+(2\times1) \end{bmatrix}$$

$$= \begin{bmatrix} -2+2+2 & 2-1+4 & 4+1+2 \\ -3+2+1 & 3-1+2 & 6+1+1 \\ -1+6+2 & 1-3+4 & 2+3+2 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 5 & 7 \\ 0 & 4 & 8 \\ 7 & 2 & 7 \end{bmatrix} \Rightarrow (A \times B).$$

$$\therefore (A \times B)' = \begin{bmatrix} 2 & 0 & 7 \\ 5 & 4 & 2 \\ 7 & 8 & 7 \end{bmatrix}$$

$\Rightarrow$ RHS : $B'A'$.

$$\hookrightarrow B' = \begin{bmatrix} -1 & 2 & 1 \\ 1 & -1 & 2 \\ 2 & 1 & 1 \end{bmatrix}$$

$$\hookrightarrow A' = \begin{bmatrix} 2 & 3 & 1 \\ 1 & 1 & 3 \\ 2 & 1 & 2 \end{bmatrix}$$

$$\hookrightarrow B'.A' = \begin{bmatrix} -1 & 2 & 1 \\ 1 & -1 & 2 \\ 2 & 1 & 1 \end{bmatrix} \times \begin{bmatrix} 2 & 3 & 1 \\ 1 & 1 & 3 \\ 2 & 1 & 2 \end{bmatrix}.$$

$$= \begin{bmatrix} (-1\times 2)+(2\times 1)+(1\times 2) & (-1\times 3)+(2\times 1)+(1\times 1) & (-1\times 1)+(2\times 3)+(1\times 2) \\ (1\times 2)+(-1\times 1)+(2\times 2) & (1\times 3)+(-1\times 1)+(2\times 1) & (1\times 1)+(-1\times 3)+(2\times 2) \\ (2\times 2)+(1\times 1)+(1\times 2) & (2\times 3)+(1\times 1)+(1\times 1) & (2\times 1)+(1\times 3)+(1\times 2) \end{bmatrix}$$

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$$= \begin{bmatrix} -2+2+2 & -3+2+1 & -1+6+2 \\ 2-1+4 & 3-1+2 & 1-3+4 \\ 4+1+2 & 6+1+1 & 2+3+2 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 0 & 7 \\ 5 & 4 & 2 \\ 7 & 8 & 7 \end{bmatrix} = B'A'.$$

$\therefore (AB)' = B'A'$.

LHS = RHS

Hence proved

4. Let the 3 numbers in GP be $\frac{a}{r}, a, ar$.

↳ **Product:**

$$\frac{a}{r} \times a \times ar = 3375.$$

$$a^3 = 3375.$$

$$a^3 = 15^3 \quad [\text{Since } 15^3 = 3375].$$

$$\boxed{\therefore a = 15}$$

↳ **Sum:**

$$\frac{a}{r} + \frac{a}{1} + \frac{ar}{1} = 65. \quad [\text{Taking LCM i.e., } r].$$

$$\frac{a + ar + ar^2}{r} = 65.$$

$$a + ar + ar^2 = 65r.$$

$$15 + 15r + 15r^2 = 65r. \quad [\text{Substituting 'a' value}].$$

$$15 + 15r + 15r^2 - 65r = 0.$$

$$15 - 50r + 15r^2 = 0. \quad \boxed{\div \text{ by } 5}$$

$$3 - 10r + 3r^2 = 0.$$

$$3r^2 - 10r + 3 = 0. \qquad 9r^2$$

$$3r^2 - 9r - 1r + 3 = 0. \qquad -9r - 1r = -10r$$

$$3r(r-3) - 1(r-3) = 0.$$

$$(3r-1) = 0 \ \& \ (r-3) = 0.$$

$$3r = 1 \qquad \boxed{r = 3}$$

$$\boxed{r = \frac{1}{3}}$$

↳ Substituting $a$ & $r$ values:

(i) $a = 15$ & $r = \frac{1}{3}$

1) $\frac{a}{r} = 15 \div \frac{1}{3} = 15 \times \frac{3}{1} = \boxed{45}$.

2) $a = \boxed{15}$.

3) $ar = \cancel{15}^{5} \times \frac{1}{\cancel{3}_1} = \boxed{5}$

(ii) $a = 15$ & $r = 3$.

1) $\frac{a}{r} = \frac{15}{3} = \boxed{5}$

2) $a = \boxed{15}$

3) $ar = 15 \times 3 = \boxed{45}$

∴ The 3 numbers in GP are 45, 15 & 5 or 5, 15 & 45

Verification:

| Sum = 65 | Product = 3375 |
|---|---|
| 5+15+45 = 65 | 5×15×45 = 3375 |
| 65 = 65 | 3375 = 3375 |

2. a)

Initial / Starting salary = 3,60,000.

Second year salary = 3,96,000.

$3,60,000 \times 10/100 + 36,000 = 3,96,000$

3,60,000 , 3,96,000 ......

$a = 3,60,000$ , $r = \frac{T_2}{T_1} = \frac{396000}{360000} = \boxed{1.1}$ , $T_5 = ?$, $S_5 = ?$, $n = 5$

↳ $T_5$ (5th year salary).

$T_n = a \cdot r^{n-1}$

$T_5 = 360000 \times (1.1)^{5-1}$

$= 3,60,000 \times (1.1)^4$

$= 3,60,000 \times 1.46$

$= \boxed{₹5,25,600}$

↳ $S_5$ (Total salary in 1st 5 years).

$S_n = \frac{a(r^n - 1)}{r-1}$

$S_5 = \frac{3,60,000 ((1.1)^5 - 1)}{1.1 - 1}$

$= \frac{3,60,000 \, (1.61-1)}{0.1}$

$= \frac{3,60,000 \times 0.61}{0.1}$

$= \boxed{₹ 21,96,000}$

∴ His salary in the 5th year was ₹ 5,25,600/-
Total salary in first 5 years was ₹ 21,96,000.

b) 500, 510, 520 . . . . . . 1,000.

$a = 500$, $d = T_2 - T_1 = 510 - 500 = 10$, $T_n = 1000$, $S_n = ?$, $n = ?$

↳ $T_n = a + (n-1)d$

$1000 = 500 + (n-1)\,10.$

$1000 = 500 + (n-1)\,10.$

$1000 - 500 = 10(n-1).$

$500 = 10(n-1).$

$\frac{500}{10} = n - 1.$

$50 = n - 1.$

$\therefore n = 50 + 1 \Rightarrow \boxed{51}$

↳ $S_{51}$.

$S_n = \frac{n}{2}\left[2a + (n-1)d\right].$

$S_{51} = \frac{51}{2}\left[2 \times 500 + (51-1)10\right].$

$= 25.5\,[1000 + 50 \times 10].$

$= 25.5\,[1000 + 500].$

$= 25.5 \times 1500.$

$= \boxed{38,250.}$

∴ The sum of all natural numbers between 500 & 1000, which are divisible by 10 is 38,250.

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28/7/22

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Part - B

7.

$$A = \begin{bmatrix} 8 & -7 & -3 \\ 4 & 17 & -11 \\ 13 & -9 & 2 \end{bmatrix}$$

a) $A + A'$.

$\hookrightarrow A' = \begin{bmatrix} 8 & 4 & 13 \\ -7 & 17 & -9 \\ -3 & -11 & 2 \end{bmatrix}$

$$A + A' = \begin{bmatrix} 8 & -7 & -3 \\ 4 & 17 & -11 \\ 13 & -9 & 2 \end{bmatrix} + \begin{bmatrix} 8 & 4 & 13 \\ -7 & 17 & -9 \\ -3 & -11 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 8+8 & -7+4 & -3+13 \\ 4+(-7) & 17+17 & -11+(-9) \\ 13+(-3) & -9+(-11) & 2+2. \end{bmatrix}$$

$$= \begin{bmatrix} 8+8 & -7+4 & -3+13 \\ 4-7 & 17+17 & -11-9 \\ 13-3 & -9-11 & 2+2 \end{bmatrix} \Rightarrow \begin{bmatrix} 16 & -3 & 10 \\ -3 & 34 & -20. \\ 10 & -20 & 4 \end{bmatrix} \Rightarrow A + A'.$$

b) $A' - A = \begin{bmatrix} 8 & 4 & 13 \\ -7 & 17 & -9 \\ -3 & -11 & 2 \end{bmatrix} - \begin{bmatrix} 8 & -7 & -3 \\ 4 & 17 & -11 \\ 13 & -9 & 2 \end{bmatrix}$

$$= \begin{bmatrix} 8-8 & 4-(-7) & 13-(-3) \\ -7-4 & 17-17 & -9-(-11) \\ -3-13 & -11-(-9) & 2-2 \end{bmatrix}$$

$$= \begin{bmatrix} \cancel{8-8} & 4+7 & 13+3 \\ -7-4 & \cancel{17-17} & -9+11 \\ -3-13 & -11+9 & \cancel{2-2} \end{bmatrix} \Rightarrow \begin{bmatrix} 0 & 11 & 16 \\ -11 & 0 & 2 \\ -16 & -2 & 0 \end{bmatrix} \Rightarrow A' - A.$$